1 $ax^2 + bx + c = 10x^2 - 7$

$= 10x^2 + 0x - 7$

$a = 10,\ b = 0,\ c = -7$

2 $2a - b = 4$ ①

$a + 2b = -3$ ②

$4a - 2b = 8$ ③

② + ③:

$5a = 5$

$a = 1$

$a \times 1 - b = 4$

$b = -2$

3 $2a - 3b = 7$ ①

$3a + b = 5$ ②

① + 3 × ②:

$11a = 22$

$a = 2$

$3 \times 2 + b = 5$

$b = -1$

$c = 7$

4 $a(x + b)^2 + c = ax^2 + 2abx + ab^2 + c$

$a = 2$

$2ab = 4$

$b = 1$

$ab^2 + c = 5$

$2 + c = 5$

$c = 3$

5 $c(x + 2)^2 + a(x + 2) + 2 = cx^2 + 4cx + 4c + ax + 2a + d$

$c = 1$

$4c + a = 0$

$a = -4$

$4c + 2a + d = 0$

$4 - 8 + d = 0$

$d = 4$

$\therefore \quad x^2 = (x + 2)^2 - 4(x + 2) + 4$

6 $(x + 1)^3 + a(x + 1)^2 + b(x + 1) + c = x^3 + 3x^2 + 3x + 1 + ax + a + bx + b + c$

$3 + a = 0$

$a = -3$

$3 + 2a + b = 0$

$3 - 6 + b = 0$

$b = 3$

$1 + a + b + c = 0$

$c = -1$

$\therefore \quad x^3 = (x + 1)^3 - 3(x + 1)^2 + 3(x + 1) - 1$

7 $ax^2 + 2ax + a + bx + c = x^2$

$a = 1$

$2a + b = 0$

$b = -2$

$$a + c = 0$$
$$c = -1$$

8 a $$a(x+b)^3 + c = ax^3 + 3abx^2 + 3ab^2x + ab^3 + c$$
$$= 3x^3 - 9x^2 + 8x + 12$$
$$a = 3$$
$$3ab = -9$$
$$3 \times 3 \times b = -9$$
$$b = -1$$

Equating x terms:

$$3ab^2 = 8$$
$$3ab^2 = 3 \times 3 \times (-1)^2 = 9$$

The equality is impossible.

b Clearly this expression can be expressed in this form, if $a = 3,\ b = -1$ and
$$ab^3 + c = 2$$
$$-3 + c = 2$$
$$c = 5$$

9 Expanding gives the following:
$$n^3 = an^3 + 6an^2 + 11an + 6a + bn^2 + 3bn + 2b + cn + c + d$$
$$a = 1$$
$$6a + b = 0$$
$$b = -6$$
$$11a + 3b + c = 0$$
$$11 - 18 + c = 0$$
$$c = 7$$
$$6a + 2b + c + d = 0$$
$$6 - 12 + 7 + d = 0$$
$$d = -1$$

10a Expanding gives the following:
$$n^2 = an^2 + 3an + 2a + bn^2 + 5bn + 6b$$
$$a + b = 1 \qquad (1)$$
$$3a + 5b = 0 \qquad (2)$$
$$2a + 6b = 0$$
$$a + 3b = 0 \qquad (3)$$

(3) − (1):
$$2b = -1$$
$$b = -\frac{1}{2}$$
$$a + -\frac{1}{2} = 1$$
$$a = 1\frac{1}{2}$$

These do not satisfy the second equation, as $3 \times 1\frac{1}{2} + 5 \times -\frac{1}{2} = 2$.

b $$n^2 = an^2 + 3an + 2a + bn + b + c$$
$$a = 1$$
$$3a + b = 0$$
$$b = -3$$
$$2a + b + c = 0$$
$$2 - 3 + c = 0$$

$$c = 1$$
$$\therefore \quad n^2 = (n+1)(n+2) - 3(n+1) + 1$$

11a $a(x^2 + 2bx + b^2) + c = ax^2 + 2abx + ab^2 + c$

b
$$\begin{aligned} ax^2 + bx + c &= A(x+B)^2 + C \\ &= Ax^2 + 2ABx + AB^2 + C \\ A &= a \\ 2AB &= b \\ B &= \frac{b}{2a} \\ AB^2 + C &= c \\ a \times \frac{b^2}{4a^2} + C &= c \\ C &= c - \frac{b^2}{4a} \end{aligned}$$
$$\therefore \quad ax^2 + bx + x = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$$

12 $(x-1)^2(px+q) = (x^2 - 2x + 1)(px + q)$
$$= px^3 + (q - 2p)x^2 + (p - 2q)x + q$$

Equating x^3 and x^2 terms:

$$\begin{aligned} p &= a \\ q - 2p &= b \\ q - 2a &= b \\ q &= 2a + b \end{aligned}$$

Equating x and constant terms:

$$\begin{aligned} q &= d \\ p - 2q &= c \\ p &= c + 2d \end{aligned}$$

Equating the two different expressions for p and q gives:

$$\begin{aligned} d &= 2a + b \qquad (q) \\ \therefore \quad b &= d - 2a \\ a &= c + 2d \qquad (p) \\ \therefore \quad c &= a - 2d \end{aligned}$$

13 $c(x-a)(x-b) = cx^2 - acx - bcx + abc$
$$\begin{aligned} &= 3 \\ c &= 3 \\ -ac - bc &= 10 \\ -3a - 3b &= 10 \\ abc &= 3 \\ 3ab &= 3 \\ ab &= 1 \\ b &= \frac{1}{a} \\ -3a - \frac{3}{a} &= 10 \\ 3a^2 + 3 &= -10a \\ 3a^2 + 10a + 3 &= 0 \\ (3a+1)(a+3) &= 0 \end{aligned}$$
$$a = -\frac{1}{3},\ b = -3,\ c = 3$$

$$\text{or } a = -3,\ b = -\frac{1}{3},\ c = 3$$

14 $n^2 = a(n-1)^2 + b(n-2)^2 + c(n-3)^2$

$= an^2 - 2an + a + bn^2 - 4bn + 4b + cn^2 + 9c$

$a + b + c = 1$ ①

$-2a - 4b - 6c = 0$

$a + 2b + 3c = 0$ ②

$a + 4b + 9c = 0$ ③

② − ①:

$b + 2c = -1$ ④

③ − ②:

$2b + 6c = 0$

$b + 3c = 0$ ⑤

⑤ − ④:

$c = 1$

$b + 3 \times 1 = 0$

$b = -3$

$a + b + c = 1$

$a - 3 + 1 = 1$

$a = 3$

$\therefore\ n^2 = 3(n-1)^2 - 3(n-2)^2 + (n-3)^2$

15 $(x-a)^2(x-b) = (x^2 - 2ax + a^2)(x-b)$

$= x^3 - 2ax^2 - bx^2 + a^2x + 2abx - a^2b$

$-2a - b = 3$

$a^2 + 2ab = -9$

Substitute $b = -2a - 3$:

$a^2 + 2a(-2a-3) = -9$

$a^2 - 4a^2 - 6a = -9$

$-3a^2 - 6a + 9 = 0$

$a^2 + 2a - 3 = 0$

$(a+3)(a-1) = 0$

$a = -3 \text{ or } a = 1$

$b = -2a - 3$

$b = 3 \text{ or } b = -5$

Comparing the constant terms:

$c = -a^2b$

$c = (-3)^2 \times 3 = -27$

or $c = (-1)^2 \times -5 = 5$

So $a = 1,\ b = -5, c = 5$

or $a = -3,\ b = 3, c = -27$